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## ALIGNMENT OF NEMATIC LIQUID CRYSTAL ON THE SURFACE WITH SPATIAL DISTRIBUTION OF EASY AXIS AND ANCHORING ENERGY

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*In this report we consider behaviour of nematic LC on randomly aligning surface. We solved the theoretical problem on spatial distribution of director on a tested surface where easy axis lies in the azimuthal plane and is randomly distributed over the surface. We found that in combined cell angular distribution of director on the tested surface strongly depends on anchoring parameter and its dispersion. For strong anchoring the director distribution practically coincides with the easy axis distribution. Weaker anchoring results in narrowing of the admitted region of the director orientation, and emerging of maximum of the distribution function at the outermost angles of this region. The dispersion of anchoring energy over the surface leads to drastic transformations of the distribution function and shift of the maximum towards the smaller angles. We believe that results obtained will give us a powerful tool for studying LC-aligning surface interfaces.*

*Keywords:* anchoring energy; easy axis; nematic liquid crystal; random anchoring

### INTRODUCTION

Orientation of a liquid crystal (LC) on the aligning surface is characterized by the axis of easy orientation of the director and the anchoring energy [1]. Most applications require mono-domain LC cells, therefore study of LC

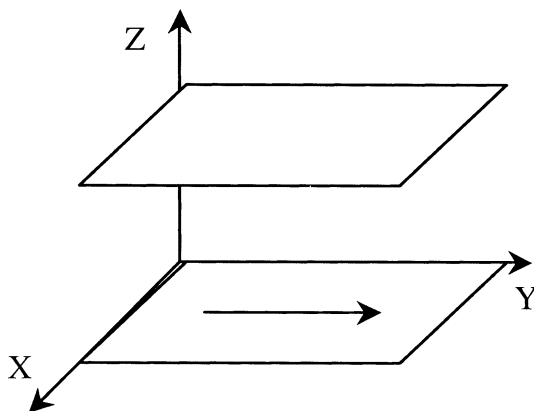
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anchoring was concentrated preferably on a nature of uniform orientation of LCs. Formation of uniform easy axis usually requires anisotropic treatment of the surface (unidirectional rubbing [2], polarized UV-irradiation [3], oblique ion-beam bombarding [4], etc.). Development of weak anchoring alignment materials, where spatial inhomogeneities of both easy axis and anchoring energy are essential, raised interest to study of LC orientation on untreated or weakly treated surfaces [5–7]. In this report we consider behaviour of nematic LC on randomly aligning surface. We have solved a problem on spatial distribution of the director of nematic LC on a tested surface where the easy axis lies in the azimuthal plane and is randomly distributed over the surface. We found that in combined cell, made from strongly anchoring reference surface with a planar easy axis and tested surface, the distribution of the director on the tested surface strongly depends on the anchoring parameter  $\xi = WL/K$  ( $W$  is anchoring energy of LC on tested surface,  $L$  – cell thickness,  $K$  – elastic constant) and its dispersion over the surface. It was shown that for  $\xi > \pi/2$  there is one-to-one correspondence between easy-axis and director orientation and the director follows the easy axis “lagging behind”. For  $\xi < \pi/2$  the one-to-one correspondence vanishes and given director orientation corresponds to two different easy axes. For a strong anchoring ( $\xi > 10$ ) the director distribution practically coincides with the distribution of the easy axis. Decreasing of the anchoring results in narrowing of the admitted region of the director orientation, and emerging of maximum of the distribution function at the outermost angles of this region. The dispersion of the anchoring energy over the surface leads to drastic transformations of the distribution function and shift of the maximum towards the smaller angles. We believe that results obtained will give us a powerful tool in experimental studies of LC-aligning surface interfaces.

## EASY AXES ARE RANDOMLY DISTRIBUTED OVER THE SURFACE

Consider combined cell – bottom reference substrate (strong unidirectional anchoring) and upper tested substrate (random anchoring, i.e. distribution function of easy axes orientation has the form  $f_0(\phi_0) = 1/\pi$ ). Since anchoring on the reference substrate is assumed to be infinite, a director distribution in a cell is determined by anchoring parameter  $\xi \equiv WL/K$ , where  $K$  – elastic constant of LC,  $W$  – anchoring energy at the tested substrate. At  $\xi \rightarrow 0$  the director all over the cell lies parallel to the easy axis on the reference surface., while infinite  $\xi$  causes the director to visualise spatial distribution of the easy axis on the tested surface. In the latter case a multi-domain structure is formed by LC on the randomly aligning surface.



**FIGURE 1** Geometry of the cell and reference frame selection.

Let us introduce Cartesian co-ordinate frame, as is depicted in Figure 1. Free energy of the nematic liquid crystal confined between the reference and random surfaces is given by the following terms

$$F = F_V + F_{S_1} + F_{S_2},$$

where

$$F_V = \frac{1}{2} \int \left[ K_{11} (\nabla \cdot \mathbf{n})^2 + K_{22} (\mathbf{n} \cdot (\nabla \times \mathbf{n}))^2 + K_{33} (\mathbf{n} \times (\nabla \times \mathbf{n}))^2 \right] dV$$

$$F_{S_1} = -W_1 (n \cdot \mathbf{e}_1)^2$$

$$F_{S_2} = -W_2 (n \cdot \mathbf{e}_2)^2,$$

where  $W_1$ ,  $W_2$  – anchoring energies on the first and second surfaces, and  $\mathbf{e}_1$ ,  $\mathbf{e}_2$  – directions of the easy axis on the first and second surfaces.

Minimum conditions for the elastic free energy in one elastic constant approximation and with domain orientational interaction being neglected gives the Euler-Lagrange equation for the director orientation in each domain

$$\frac{d^2 \phi}{dz^2} = 0$$

and the following boundary conditions

$$\begin{aligned} \phi(z=0) &= 0, \\ \frac{d\phi}{dz} + \xi/2 \sin 2(\phi - \phi_0) \Big|_{z=L} &= 0 \end{aligned}$$

where  $\phi_0$  denotes the easy orientation direction, and  $\phi$  is a director orientation. Solution to this boundary value problem is

$$\phi = az/L,$$

where parameter  $a = \phi(z = L)$  should be found from the equation

$$2a + \xi \sin 2(a - \phi_0) = 0$$

Thus we can predict domain orientation for the given easy axis direction:

$$2\phi + \xi \sin 2(\phi - \phi_0) = 0.$$

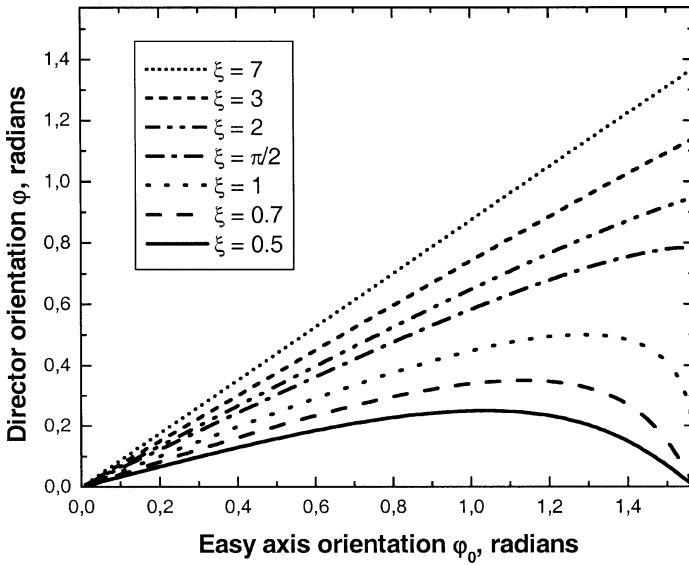
The solutions to the equation are:

$$\phi_0^1 = \phi + \text{ArcSin}(2\phi/\xi)/2$$

$$\phi_0^2 = \phi + \pi - \text{ArcSin}(2\phi/\xi)/2$$

Figure 2 gives  $\phi(\phi_0)$  dependence for different values of the anchoring parameter  $\xi = WL/K$ .

Clearly, director orientation on the tested surface strongly depends on the anchoring parameter. For  $\xi > \pi/2$  there is one-to-one correspondence



**FIGURE 2** Director orientation dependence on the easy axis direction for different anchoring parameter values.

between easy-axis and director orientation and the director follows the easy axis “lagging behind”. For  $\xi < \pi/2$  the one-to-one correspondence vanishes and two different easy axis directions lead to the same domain director orientation. For a strong anchoring ( $\xi > 10$ ) the director distribution practically coincides with the distribution of the easy axis.

To give quantitative description of liquid crystal alignment, a function of domain angular distribution at the tested surface  $f(\phi)$  is introduced. For given anchoring energy all domains with easy axes lying within  $d\phi_0^i$  around  $\phi_0^i$  will align into  $d\phi_I$  area around  $\phi_I$ , i.e.

$$f(\phi)d\phi = f_0^1(\phi_0^1)d\phi_0^1 + f_0^2(\phi_0^2)d\phi_0^2$$

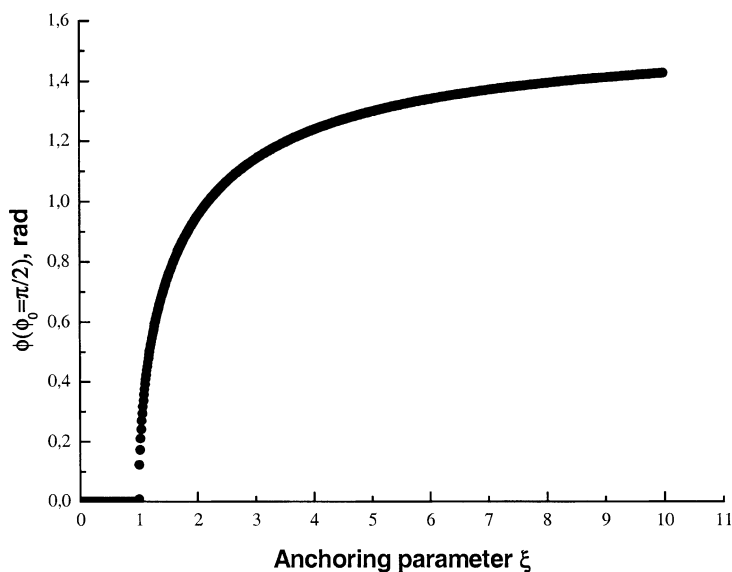
Still the second solution exists only for  $\phi > \tilde{\phi}$ :

$$f(\phi) = f_0^1(\phi_0^1) \frac{d\phi_0^1}{d\phi} + f_0^2(\phi_0^2) \frac{d\phi_0^2}{d\phi} \Big|_{\phi > \tilde{\phi}}$$

$$f_0^1(\phi_0^1) = f_0^2(\phi_0^2) = 1/\pi$$

$$\phi(\phi_0 = \pi/2) \equiv \tilde{\phi}$$

Figure 3 gives the dependence of  $\tilde{\phi}$  on anchoring parameter:



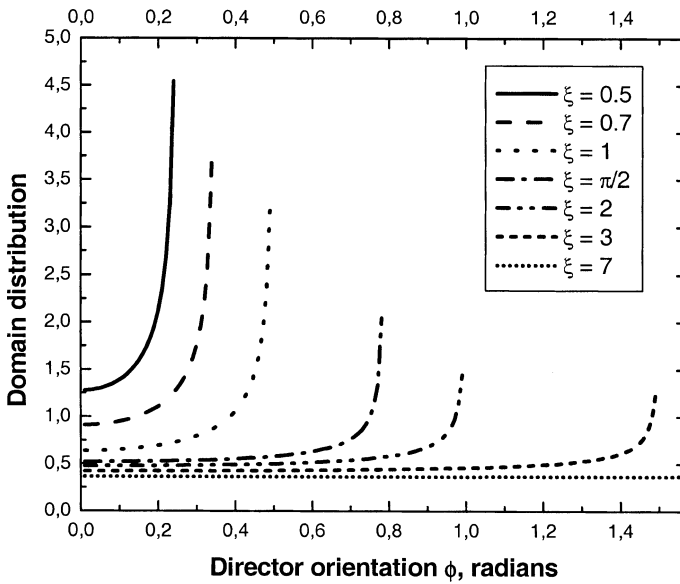
**FIGURE 3**  $\tilde{\phi}$  for different values of anchoring parameter.

Finally, distribution function reads as:

$$f(\phi) = \begin{cases} \frac{1}{\pi} \left( 1 + 1/(\xi^2 - 4\phi^2)^{1/2} \right), & \phi \in [0, \tilde{\phi}) \\ \frac{2}{\pi(\xi^2 - 4\phi^2)^{1/2}}, & \phi \in [\tilde{\phi}, \xi/2). \end{cases}$$

Figure 4 plots the distribution function for different anchoring parameters.

We found that in combined cell, made from strongly anchoring reference surface with a planar easy axis and tested surface, the distribution of the director on the tested surface strongly depends on the anchoring parameter  $\xi = WL/K$  ( $W$  is anchoring energy of LC on tested surface,  $L$  - cell thickness,  $K$  - elastic constant) and its dispersion over the surface. It was shown that for  $\xi > \pi/2$  there is one-to-one correspondence between easy-axis and director orientation and the director follows the easy axis “lagging behind”. For  $\xi < \pi/2$  the one-to-one correspondence vanishes and given director orientation corresponds to two different easy axes. For a strong anchoring ( $\xi > 10$ ) the director distribution practically coincides with the distribution of the easy axis. Decreasing of the anchoring results in narrowing of the admitted region of the director orientation, and emerging of maximum of the distribution function at the outermost angles of this region.



**FIGURE 4** Domain distribution function for different values of anchoring parameter.



## ANCHORING AND EASY AXES ARE RANDOMLY DISTRIBUTED OVER THE SURFACE

The situation considered above is rather naïve; in the reality spatial fluctuations of the parameters, which determine the value of the anchoring energy (packing and angular distribution of the aligning molecular fragments, spatial inhomogeneity of the adsorbed impurities, complex morphology of the aligning surface, etc) can be essential. We took into account this circumstances considering certain spread of the anchoring energy values.

Providing that number of domain is large enough to satisfy limitations of statistical approach. Then, for every fixed value of anchoring energy we'll have a set of domains whose easy axes are uniformly distributed in  $[-\pi/2, \pi/2]$  interval. Therefore, angular distribution of the domain orientation can be written as

$$f = \int_{\xi_{\min}}^{\xi_{\max}} f_{\xi} \cdot \rho(\xi) d\xi,$$

where  $f_{\xi}$  denotes the distribution function for the constant anchoring parameter  $\xi$ , and  $\rho(\xi)$  – is a distribution of domains on the anchoring parameter.

Substituting expression for distribution function obtained for the constant anchoring parameter gives the analytical expression for angular distribution of the domains with the dispersion of anchoring parameter:

$$f(\phi) = \frac{1}{\pi} \cdot \left[ \int_{\xi_{\min}}^{\xi_{\max}} \left( 1 + 1/(\xi^2 - 4\phi^2)^{1/2} \right) \rho(\xi) d\xi + \int_{\xi_{\min}}^{\xi_{\text{crit}}(\phi)} 2/(\xi^2 - 4\phi^2)^{1/2} \rho(\xi) d\xi \right]$$

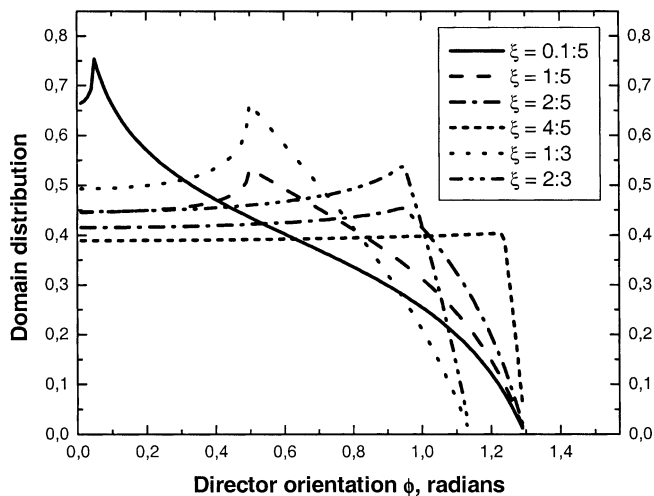
In case of uniform distribution of anchoring parameter over the  $[\xi_{\min}, \xi_{\max}]$  interval, i.e. distribution is rectangular-shaped

$$f(\phi) = 1/(\xi_{\max} - \xi_{\min}) \int_{\xi_{\min}}^{\xi_{\max}} f_{\xi}(\phi) d\xi$$

To evaluate this integral for each  $\phi$  the critical value of anchoring parameter  $\xi_{\text{crit}}$  is found, and finally distribution function presented on Figure 5 are obtained.

## CONCLUSIVE REMARKS

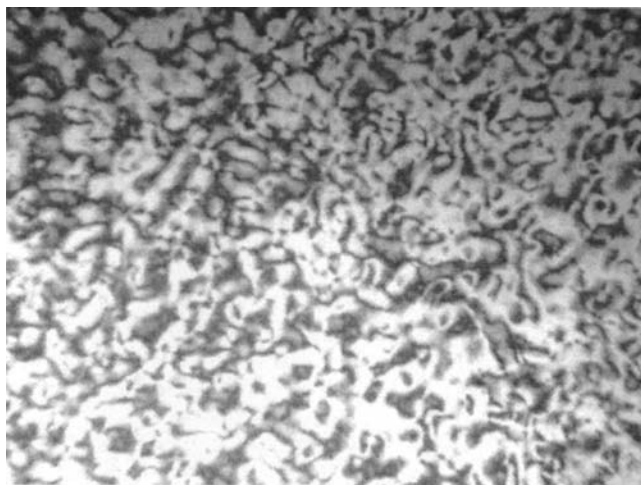
The obtained distribution functions look rather specific: the presence of sharp maxima, limiting angles for director orientation on the tested surface,



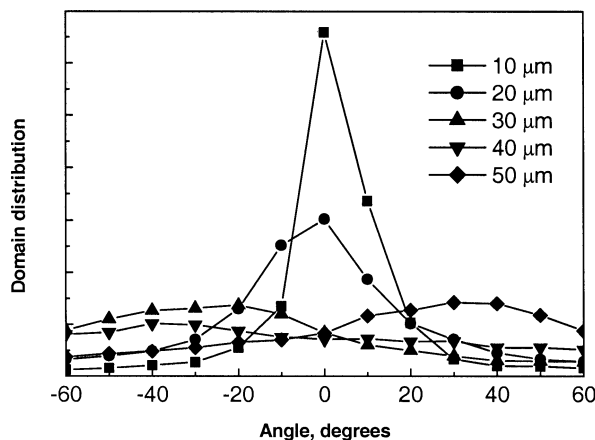
**FIGURE 5** Distribution function for different sets of anchoring parameter  $\xi$ .

minimal quantity of domains with the director orientation on the tested surface parallel to the director orientation on the reference surface had to result in unusual LC textures on a randomly aligning surfaces.

At the same time, our tentative preliminary experimental results show ordinary chaotic textures (Fig. 6) and routine distribution function with a maximum along the director at the reference surface (Fig. 7).



**FIGURE 6** Texture of liquid crystal on randomly aligning surface in polarising microscopy.



**FIGURE 7** Angular distribution of domain alignment for different cell thickness.

These results indicate that layer of LC molecules adsorbed by the polymer determines the angular distribution of the director on the tested surface. We believe that to get the distribution functions, which correspond to the obtained theoretically, we need to find a memory free aligning material. We assume that diamond like carbon (DLC) films are promising candidates for memory free surfaces [8].

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